IX. *On the laws which regulate the polarisation of light by re-flexion from transparent bodies.* By David Brewster, *LL.D. F.R.S. Edin. and F.S.A. Edin. In a letter addressed to the Right Hon. Sir* Joseph Banks, *Bart. K.B.P.R.S.*

Read March 16, 1815.

Dear Sir,

The discovery of the polarisation of light by reflexion, con-stitutes a memorable epoch in the history of optics; and the name of Malus, who first made known this remarkable pro-perty of bodies, will be for ever associated with a branch of science which he had the sole merit of creating. By a few brilliant and comprehensive experiments he established the general fact, that light acquired the same property as one of the pencils formed by double refraction, when it was reflected at a particular angle from the surfaces of all transparent bodies: he found that the angle of incidence at which this property was communicated, was greater in bodies of a high refractive power, and he measured, with considerable accuracy, the polarising angles for glass and water. In order to discover the law which regulated the phenomena, he com-pared these angles with the refractive and dispersive powers of glass and water, and finding that there was no relation be-tween these properties of transparent bodies, he draws the following general conclusion. " The polarising angle neither " follows the order of the refractive powers, nor that of the

" dispersive forces. It is a property of bodies independent
" of the other modes of action which they exercise upon
" light."

This premature generalisation of a few imperfectly ascer-
tained facts, is perhaps equalled only by the mistake of Sir
ISAAC NEWTON, who pronounced the construction of an
achromatic telescope to be incompatible with the known prin-
ciples of optics. Like NEWTON, too, MALUS himself aban-
doned the enquiry; and even his learned associates in the
Institute, to whom he bequeathed the prosecution of his views,
have sought for fame in the investigation of other properties
of polarised light.

In the summer of 1811, when my attention was first turned
to this subject, I repeated the experiments of MALUS, and
measured the polarising angles of a great number of trans-
parent bodies. I endeavoured, in vain, to connect these results
by some general principle: the measures for *water* and the
precious stones afforded a surprising coincidence between the
indices of refraction and the tangents of the polarising angles;
but the results for glass formed an exception, and resisted
every method of classification. Disappointed in my expecta-
tions, I abandoned the enquiry for more than twelve months,
but having occasion to measure the polarising angle of *topaz*,
I was astonished at its coincidence with the preceding law,
and again attempted to reduce the results obtained from *glass*
under the same principle. The piece which I used had two
surfaces excellently polished. The polarising angle of one
of these surfaces almost exactly accorded with the law of the
tangents, but with the other surface there was a deviation of
no less than two degrees. Upon examining the cause of this

anomalous result, I found that one of the surfaces had suf-
fered some chemical change, and reflected less light than any
other part of the glass. This artificial substance acquires
an incrustation, or experiences a decomposition by exposure
to the air, which alters its polarising angle without altering its
general refractive power. The perplexing anomalies which
Bouguer observed in the reflective power of plate glass, were
owing to the same cause, and so liable is this substance to
these changes, that by the aid of heat alone, I have produced
a variation of 9° on the polarising angle of flint glass, and
given it the power of acting upon light like the coloured
oxides of steel.

Having thus ascertained the cause of the anomalies pre-
sented by glass, I compared the various angles which I had
measured, and found that they were all represented by the
following simple law.

The index of refraction is the tangent of the angle of polarisation.

In the course of last summer, when I had the pleasure of
seeing M. Arago, I mentioned to him the relation which I
had discovered between the refractive powers, and the tan-
gents of the polarising angles. He informed me, that he had
found the polarising angle of air to be 45° or 47°, which being
at the very extremity of the scale would afford a good test
of the accuracy of the law. Now, if we take the refractive
power of air at 1.00031 the polarising angle will be 45° 0′ 32″,
a result which agrees most strikingly with the observed
angle.

In the following table I have given the polarising angles
of eighteen transparent bodies, as determined by experiment,
and as deduced from the law of the tangents. I have added in

the *fourth* column the differences between the calculated and observed angles, and in the *fifth* column the calculated angles of polarisation for the second surfaces of the bodies subjected to experiment.

Table containing the calculated and observed polarising angles for various bodies.

Names of the Bodies..	Calculated polarising angles for the *first* surface.			Observed polarising angles for the *first* surface.		Difference between the calculated and observed angles.		Calculated polarising angles for the *second* surface.		
	°	′	″	°	°			°	′	″
Air　-　-　-　-　-　-	45	0	32	45 or	47			44	59	28
Water　-　-　-　-	53	11	0	52°	45′	0° 26′	—	36°	49′	
Fluor spar　-　-　-	55	9	0	54	50	0 19	—	34	51	
Obsidian　-　-　-	56	6	0	56	3	0 3	—	33	54	
Birdlime　-　-　-　-	56	40	0	56	46	0 6	+	33	20	
Sulphate of lime　-	56	45	0	56	28	0 17	—	33	15	
Rock crystal　-　-	56	58	0	57	22	0 24	+	33	2	
Opal coloured glass	58	33	0	58	1	0 32	—	31	27	
Topaz　-　-　-　-	58	34	0	58	40	0 6	+	31	26	
Mother of pearl　-	58	50	0	58	47	0 3	—	31	10	
Iceland spar　-　-	58	51	0	58	23	0 28	—	31	9	
Orange coloured glass	59	28	0	59	12	0 16	—	30	32	
Spinelle ruby　-　-	60	25	0	60	16	0 9	—	29	35	
Zircon　-　-　-　-	63	0	0	63	8	0 8	+	27	0	
Glass of antimony	64	30	0	64	45	0 15	+	25	30	
Sulphur　-　-　-　-	63	45	0	64	10	0 25	+	26	15	
Diamond　-　-　-	68	1	0	68	2	0 1	+	21	59	
Chromate of Lead	68	3	0	67	42	0 21	—	21	56	

The coincidence between the calculated and observed angles, as shown in the preceding Table, must appear very remarkable to those who are aware of the difficulty of measuring

correctly the index of refraction for the *mean* refrangible ray, and the still greater difficulty of determining the angle at which the intensity of the evanescent pencil is a minimum. The total amount of the errors in seventeen observations is 259 minutes, which gives an average error of 15′ for each observation.   In general the observed angles are less than the calculated angles, the number of negative being to the number of positive differences as 174′ is to 85′.

This circumstance arises from two separate causes, which ought to be carefully kept in view in all experiments on the polarising angles of bodies.

1. In order to illustrate the first of these causes, let us take the case of *Zircon*, in which the intensity of the evanescent pencil is a minimum at 63° of incidence. At 64° the intensity of the pencil which vanishes at 63° is much greater than that of the pencil at 62° on account of its falling more obliquely upon the reflecting surface, and consequently the intensity of that pencil varies more rapidly in passing from 64° to 63° than from 62° to 63°.   Hence, in determining the point of minimum intensity, it is more likely, from the way in which the observation must be made, that the observed angle will fall *below* than above the real polarising angle.

2. As the differently coloured rays have different angles of polarisation, and as the most luminous rays of the spectrum have less refractive power than the mean refrangible rays, the observed polarising ought always to be less than the polarising angle for the mean rays. Hence, all the observed angles in the preceding Table ought to be increased by a certain quantity, or, what is the same thing, the index of refraction for the most luminous rays ought to be employed instead

of the mean index of refraction in computing the first column.

The law of the polarisation of light by reflexion being thus experimentally established, we shall now proceed to point out its geometrical consequences, and to arrange, under separate propositions, the new truths to which it leads, as well as those which I have obtained from direct experiment. It will thus be seen, that the subject assumes a scientific form, and that we can calculate *a priori*, the result of every experiment, whether the light is incident upon the first or second surface of transparent bodies, or upon the separating surface of different media, or whether it undergoes a series of successive reflexions in the same plane, or in planes at right angles to each other.

SECT. I. *On the laws of the polarisation of light, by reflexion from the first surfaces of transparent bodies.*

PROP. I.

When a pencil of light is incident upon a transparent body at an angle, whose tangent is equal to the index of refraction, the reflected portion will be either wholly polarised, or the quantity of polarised light which it contains will be a maximum.

This proposition is a repetition of the general law already established. In *water, glass*, and other bodies, whose refractive power is less than 1.6, almost the whole of the pencil is polarised, at the polarising angle; but in diamond, realgar, chromate of lead, oil of cassia, &c. whose refractive power exceeds 1.6, the whole of the reflected pencil does not suffer polarisation, but the quantity of polarised light is a *maximum* at the angle indicated in the proposition.   See Sect. V.

PROP. II.

The differently coloured rays which compose a beam of white light are polarised at angles of reflexion whose tangents are equal to their respective indices of refraction.

This Proposition might have been admitted as a corollary to Prop. I.; but I have established it by the following experiment. A ray of light incident upon *oil of cassia* at an angle of 58° 38′, suffers its maximum polarisation. When the angle is 57° 38′ the *blue* rays predominate in the pencil that approaches to evanescence, while at an angle of 59° 38′ the *red* rays predominate. Hence it follows, that the polarisation of the red light is a maximum at an angle below the mean polarising angle, and the polarisation of the blue light a maximum at an angle, above the mean polarising angle. See Sect. **V.**

PROP. III.

When the refractive power of any body is infinitely small, its polarising angle will be 45°.

The limit to which the index of refraction constantly approaches is 1.000 which is the tangent of 45°

PROP. IV.

When a pencil of light is polarised by reflexion, the sum of the angles of incidence and refraction is a right angle.

Let MN, (Fig. 1, Pl. VI.) be the reflecting surface, and BA, a ray of light polarised by reflexion in the direction AD, and let AC be the refracted ray. Then since EF, the tangent of the polarising angle BAE, is equal to m, or the index of refraction, we have by the law of the sines, $CL = \dfrac{BG}{m} = \dfrac{BG}{EF}$. But from the

similar triangles ABH, AEF, we have AH or BG : HB :: EF : Rad.; and HB $= \frac{BG}{EF}$, consequently CL $=$ HB and the angle BAN $=$ CAK. But EAB $+$ BAN $= 90°$, hence EAB $+$ CAK $= 90°$.

Cor. The complement of the polarising angle is equal to the angle of refraction.

PROP. V.

When a ray of light is polarised by reflexion, the reflected ray forms a right angle with the refracted ray.

Since the angles DAM, BAN, CAK (Fig. 1.) are equal to one another, the angle DAC is equal to the right angle MAK: hence the reflected ray AD forms a right angle with the refracted ray AC.

PROP. VI.

If light were polarised simply by the action of the reflecting force, the polarising angle would be 45°.

For when the refracting force is infinitely small, the polarising angle is 45°. The reflecting force is also infinitely small in this case, but any diminution of the reflecting force, however great, does not alter the direction of the reflected ray with respect to the incident ray, or the position of any point or side of a ray with regard to the direction of its motion. It may also be remarked that 45° is the only angle of reflexion at which any point or side of a ray makes a revolution of 90° relative to the direction of its motion. See Prop. VIII, and Cor. 2, of the same Proposition.

PROP. VII.

Every ray of light polarised by reflexion has been acted upon by the refracting force before it has suffered reflexion.

This follows from the light not being polarised at 45°, but at various angles increasing with the refracting force.

Cor. It results from this Proposition, that light suffers a partial refraction before it is reflected; and that the refractive force extends to a greater distance than the reflecting force from the surface of transparent bodies. This result is not only consistent with the most extensive analogies, but affords an explanation of phenomena, which have hitherto been unex-plained.

BOUGUER, for example, observed that at $87\frac{1}{2}°$ of incidence, a surface of water reflected 614 rays, while glass reflected only 584. Now supposing the light to be refracted by the water and the glass, before it suffers reflexion, the real angle of incidence upon the glass will be only 57° 44', while the angle of incidence upon the water will be 61° 5'; so that the pencil being incident more obliquely upon the water, ought to be more copiously reflected.

PROP. VIII.

When a ray of light is incident at the polarising angle upon any substance whatever, it receives such a change in its direction, by the action of the refracting force, that the real angle of incidence at which it is reflected and polarised is 45°

Let MN, Fig. 2, be the refracting and the reflecting surface, and OP the termination of the sphere of refracting activity. Let a ray RG be incident at G, at the polarising angle, and

let it be refracted into the line GB, before it is reflected from the surface MN.* A part of the ray GB will penetrate the surface MN, and be refracted into the line BF, while another part will be reflected in the direction BA, and again refracted at A into the line AS. Continue SA to C and FB to D. Then since half of the refraction is supposed to be performed before the ray reaches B, and half of it after it penetrates the medium MN, we have BAC $=$ DBC $=$ half the angle of deviation. But by Prop. V, ADB is a right angle, hence ABC is also a right angle, and the angles ABE, GBE, each half a right angle, or 45°.

Cor. 1. At the instant of reflexion, when the refraction at B commences, the refracted ray sets off at right angles to the reflected portion.

Cor. 2. The *real angle of polarisation* is 45°, the effect of the refractive force being merely to bend the ray of light so as to make it suffer reflexion at this particular angle.

Cor. 3. The excess of the angle formed by the incident and the polarised ray, above a right angle, is equal to the angle of deviation. The angle PAB, Fig. 1, which is equal to the angle of deviation OAC, is obviously the excess of DAB above the right angle DAP.

Sect. II. *On the laws of the polarisation of light by reflexion from the second surfaces of transparent bodies.*

When a ray of light is incident upon a parallel plate of

* In order to keep the figure from being complicated, I have supposed the reflexion to take place all at once when the ray reaches the surface MN. The demonstration would have been exactly the same if the ray had been represented as suffering a gradual reflexion in passing through the sphere of reflecting activity.

any transparent body, the image reflected by the first surface is nearly coincident with the image reflected by the second surface, and Malus observed that they were both polarised at the same time. As the angles at which the rays are incident upon the two surfaces are very different, this result appeared quite inexplicable; but it will be seen from the following Propositions, that the simultaneous polarisation of the two pencils is a necessary consequence of the general law, and derives from that law the most satisfactory explanation.

PROP. IX.

When a pencil of light is incident on the second surface of transparent bodies, at an angle whose co-tangent is equal to the index of refraction, the reflected portion will be either wholly polarised, or the quantity of polarised light which it contains will be a maximum.

As the images formed by the first and second surfaces of a transparent plate are simultaneously polarised, this Proposition is established by the experimental results in the preceding Table.

PROP. X.

The angle of polarisation at the second surface of transparent bodies, is the complement of the angle of polarisation at the first surface.

As the angle of incidence at the second surface is equal to the angle of refraction at the first surface, and as this latter angle is, by the Corollary to Prop. IV. equal to the complement of the angle of polarisation, it follows, that the two polarising angles are complementary to each other.

PROP. XI.

When a ray of light is polarised by reflexion from the second surface of transparent bodies, the reflected ray will form a right angle with the refracted ray.

Let AB, Fig. 3, be a ray incident at the first surface MN, AD the ray polarised at that surface, AC the ray incident at the second surface PQ, and CM the ray polarised at that surface; then if CF be the refracted ray, the angle MCF is a right angle. By Prop. V, DAC is a right angle, and on account of the parallelism of MN, PC, and BA, CF, the angle FCP is equal to DAM, but MCP is equal to MAC, hence the whole MCF is equal to the whole DAC, or a right angle.

Cor. 1. The ray CM, reflected by the second surface, is at right angles to the ray AB incident on the first surface.

Cor. 2. The internal reflected ray CM forms with the external reflected ray AD, an angle equal to the angle of deviation CAO.

Cor. 3. The ray CF emerging from the second surface forms with the first reflected ray, AD an angle equal to the complement of the angle of deviation.

PROP. XII.

When a ray of light is incident at the polarising angle, upon the second surface of transparent bodies, it receives such a change in its direction from the action of the refracting force, that the real angle of incidence, at which it is reflected and polarised, is 45°

By the very same reasoning which was used in Prop. XI,

it may be shown that the angle ABC, *fig.* 4, is a right angle; but BC being a continuation of BG, ABG will also be a right angle, and consequently the angle of incidence EBG will be half a right angle, or 45°.

<div align="center">Prop. XIII.</div>

When a ray of light is incident on the second surface of a transparent body at an angle whose sine is greater than the reciprocal of the index of refraction, or at an angle greater than the angle of total reflexion, the reflected light will consist of two pencils, one of which is polarised in the plane of reflexion, and the other in a plane perpendicular to the plane of reflexion.

The experiments by which I ascertained this singular property were conducted in a manner similar to those of Malus upon polished metals. A ray of light moving horizontally in the direction of the meridian, after having been polarised in the plane of the horizon, was made to fall upon the second surface of a transparent body facing the south-east or south-west, and inclined at such an angle to the horizon, as to receive the ray near the limit of total reflexion. The polarised ray was depolarised by the action of the second surface, so that the images of the object from which it proceeded, when viewed through a prism of calcareous spar, continued visible in every part of its revolution, an effect which could only be produced by the power of the second surface to form two oppositely polarised images.

When the plane of the second reflexion is either parallel, or perpendicular to the plane in which the ray was originally polarised, the ray will suffer no change by the second reflexion, one of the images formed by a prism of calcareous spar vanishing in every quadrant of its circular motion.

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In order to ascertain the relation between the polarising angle at the second surface of transparent bodies, and the angle at which they reflect the whole of the incident pencil, let us make.

$$\text{The index of refraction} = m$$
$$\text{Sine of the angle of total reflexion } \frac{1}{m}$$
$$\text{Cotangent of the polarising angle } m$$
$$\text{Tangent of the polarising angle } \frac{1}{m}$$

Since the sine of any angle is always less than the tangent, the polarising angle whose tangent is $\frac{1}{m}$ will always be less than the angle of total reflexion whose sine is $\frac{1}{m}$. The angle of polarisation, therefore, must fall without the limit of total reflexion, but it will gradually approach to that limit as the refractive power increases. When the pencil, however, is incident within the sphere of total reflexion, the quantity of polarised light is so near its maximum, that the experiments can be conducted with almost the same result, as if the polarising angle had exceeded the angle of total reflexion. The only consequence of the difference between the two angles is, that the depolarised image is inferior in point of intensity to the other image.

The following measures for *flint glass* with a refractive power of 1.600, and of *diamond* with a refractive power of 2.80, will show the relations between these two angles.

			°	′
Polarising angle for the second surface of *flint glass*			32	0
Angle at which total reflexion commences	-		38	45
	Difference		6°	45′

Polarising angle for the second surface of *diamond*   21 59

Angle at which total reflexion commences   -   23 46

                                    Difference   1° 47'

SECT. III. *On the laws of the polarisation of light by reflexion from the separating surfaces of different media.*

Although the attention of MALUS was directed to this branch of the subject, yet he does not appear to have obtained even a single measure of the angles at which light is polarised at the separating surfaces of different media. " After having de- " termined, " he observes," the angles under which polarisa- " tion takes place with respect to different bodies, *water* and " *glass*, for example, I sought for that at which the same " phenomenon would take place at their surface of separation, " when they are in contact, but the law according to which " this last angle depends upon the first two still remains to " be discovered." I have often attempted the same experi- ment with the same want of success, but besides being unable to determine the *law*, I could never find that there was any maximum angle of polarisation at the common surface of water and glass, when the light was incident from air. It is curious to remark, that MALUS does not say, that such an angle existed although this may be considered as implied in the observation that the law still remains to be determined. Now it is sufficiently singular, as will appear from the follow- ing propositions, that there is no angle of incidence at the first surface of the water which will admit the light to be pola- rised at the common surface of the water and the glass.

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Prop. xiv.

When a pencil of light is incident upon the separating surface of two media having different indices of refraction m m', it will be polarised at an angle whose tangent is equal to the quotient of the greater index of refraction divided by the lesser, or $\frac{m}{m'}$.

This Proposition is a necessary consequence of the general law, and is also deduced from direct experiment.

If we call A the angle whose tangent is equal to $\frac{m}{m'}$, then the corresponding angle at which the pencil is incident from air upon the first surface of the upper medium, or $\alpha = \text{sin. A} \times m$.

In the case of *water* and *glass*, where m is equal to 1.525, and m' to 1.336, we have the polarising angle at the surface of separation, or A = 48° 47', and sin. A $\times$ m' = 1.0048, consequently α is greater than 90°. Hence it follows, that when a ray of light is incident upon a parallel plate of water lying upon a plate of glass, there is no angle of incidence upon the first surface of the water at which the ray will suffer polarisation at the separating surface of the two media. The polarisation of the incident pencil increases from 0° to 90°, and is nearly complete at 90°.

When m is equal to 1.508, which is sometimes the case, then sin. A $\times$ m' = 1.000, and α = 90° exactly.

This conclusion was so unexpected that I immediately endeavoured to confirm it by experiment. The result was exactly conformable to the law : the polarisation of the pencil became more and more perfect from 0° to 90° of incidence. Between 80° and 90° the change was scarcely perceptible, owing to the slow variation of the sines, for when the pencil is

incident at 80°, the angle of incidence at the separating surface
is 47° 29′, while at an incidence of 90° it is no more than
48° 28′, differing only 59′ from the other.

PROP. XV.

*When light is polarised at the separating surface of two media, the
sum of the angles of incidence and refraction is a right angle, and
the reflected ray forms a right angle with the refracted ray.*

This proposition is demonstrated in the same manner as
Prop. IV and V, the separating surface producing always the
same phenomena as the first surface of any body, whose index
of refraction is equal to the quotient of the indices of refrac-
tion for the two contiguous bodies.

It would be a waste of time to extend the application of the
general law to other cases where the reflecting surfaces are
inclined at different angles, or where the incident pencil tra-
verses a number of different media, and receives particular
changes at each successive reflexion. We shall, therefore, go
on to another branch of enquiry, and consider the laws which
regulate the phenomena when a pencil is polarised by several
successive reflexions, a subject to which MALUS has not even
alluded.

SECT. IV. *On the law of the polarisation of light by successive*
reflexions.

PROP. XVI.

When a ray of light is incident at any angle except a right angle
  upon the surface of a transparent body, a certain portion of the
  reflected light is completely polarised, while the remaining por-
  tion has suffered a physical change, or has acquired, in various
  degrees, a character approaching to complete polarisation.

This proposition has been established by direct experiments
made with glass, whose polarising angle is 56° 45'.

If a pencil of light is reflected from glass at an angle of
62° 30', or 50° 20', i. e. either above or below the polarising
angle, the portion of light which is not completely polarised,
has so far received this character, that it will be completely
polarised by a second reflexion at the same angle, whereas
had it been absolutely unpolarised light, it could not have
been polarised at any angle different from 56° 45', the real
angle of polarisation.

In like manner *three* reflexions at an angle of 65° 33' or
46° 30', or *four* reflections at an angle of 67° 33', or 43° 51'
will polarise the whole pencil, while at angles above 82° or
below 18° more than 100 reflexions are necessary to produce
complete polarisation.

The truth of the proposition for the transmitted rays is
established by the experiments which I have already published
on the polarisation of light by oblique refraction.* If a pencil

* See *Phil. Trans.* 1814, Part I. p. 219.

of light is polarised by twenty-four plates at an angle of 60°, then it is obvious that twelve plates will not polarise the whole pencil at the same angle.

Let us suppose that the portion not polarised amounts to twenty rays out of a hundred. Then, if these twenty rays were absolutely unpolarised, and in the same state as direct light, they would require to pass through twenty-four plates at an angle of 60°, in order to be polarised: but the experiments show that they require only to pass through other twelve plates at that angle. It therefore follows, that the twenty rays have been half polarised by the first twelve plates, and the polarisation completed by the other twelve. Hence we see the mistake of Malus, who observes, that the light transmitted obliquely through glass consists 1st. of a quantity of polarised light, and " 2d. *of another portion not modified,* " *and which preserves the characters of direct light.*"

PROP. XVII.

If a ray of light is partly polarised by reflexion at any angle, it will be more and more polarised by every successive reflexion in the same plane, till its polarisation is complete, whether the reflexions are made at angles all above or all below the polarising angle, or at angles some of which are above and some below the polarising angle.

This proposition is deduced from numerous experiments which may be easily repeated. It is extremely difficult, however, on account of the rapid attenuation of the light when it has undergone a few reflexions from glass, to determine satisfactorily the relation between the number of reflexions and the angles of incidence at which they polarise a pencil of light. The experiments which I have made are represented by the

following law, A, *a* being the angles of incidence above and below the polarising angle, *m* the index of refraction, and N, *n* the number of reflexions above or below the polarising angle.

$$\text{Tang. A} = m \times \sqrt[3]{N}$$

when the angle of incidence is *greater* than the polarising angle for one reflexion, and

$$\text{Tang. } a = \frac{m}{\sqrt[3]{n}}$$

when the angle of incidence is less than the polarising angle for one reflexion. Hence we have

$$N = \left(\frac{\text{tang. A}}{m} \right)^3 \text{ and}$$

$$n = \left(\frac{m}{\text{tang. } a} \right)^3$$

When the successive reflections are made at different angles A, A′, A″ above the polarising angle, or *a*, *a*′, *a*″ below the polarising angle, the pencil will be just polarised when

$$\left(\frac{1}{\frac{\text{tang. A}}{m}} \right)^3 + \left(\frac{1}{\frac{\text{tang. A}'}{m}} \right)^3 + \left(\frac{1}{\frac{\text{tang. A}''}{m}} \right)^3 = 1. \text{ or when}$$

$$\left(\frac{1}{\frac{m}{\text{tang. } a}} \right)^3 + \left(\frac{1}{\frac{m}{\text{tang. } a'}} \right)^3 + \left(\frac{1}{\frac{m}{\text{tang. } a''}} \right)^3 = 1.$$

When some of the reflexions are made above, and some below the polarising angle, at the angles A, *a*, *a*′ A″ for example, then the pencil will be polarised when

$$\left(\frac{1}{\frac{\text{tang. A}}{m}} \right)^3 + \left(\frac{1}{\frac{m}{\text{tang. } a}} \right)^3 + \left(\frac{1}{\frac{m}{\text{tang. } a'}} \right)^3 + \left(\frac{1}{\frac{\text{tang. A}''}{m}} \right)^3 = 1.$$

When the refractive power is infinitely small, which is nearly the case in *air*, and the *gases*, we have $m = 1.000$ and

$$N = (\text{tang. A})^3, \ n = \left(\frac{1}{\text{tang. } a} \right)^3.$$ Hence when $N = n$, cotang.

$a = \text{tang. A}$, and therefore a will in this case be the complement

of A, the same effect being produced at angles equidistant from the maximum polarising angle.

In order to apply the formulæ with facility in ascertaining the number of reflexions necessary to polarise a pencil of light, I have calculated the following Table containing values of A and a for various values of N and n, the reflexions being supposed to be made from glass, whose index of refraction is equal to 1.525.

Table showing the angles at which a pencil of light is polarised by any number of reflexions at the same angle.

When the angles are *greater* than the maximum polarising angle, or 56° 45'.		When the angles are *less* than the maximum polarising angle, or 56° 45'.	
Number of reflexions necessary to polarise the incident light.	Angles at which the incident light is wholly polarised by the number of reflexions in Col. 1.	Number of reflexions necessary to polarise the incident light.	Angles at which the incident light is wholly polarised by the number of reflexions in Col. 3.
1	56 45	1	56 45
2	62 30	2	50 26
3	65 33	3	46 30
4	67 33	4	43 51
5	69 1	5	41 43
6	70 9	6	40 0
7	71 5	7	38 33
8	71 51	8	37 20
9	72 30	9	36 15
10	73 4	10	35 18
27	77 40	27	26 39
64	80 41	64	20 52
100	81 57	100	18 11
125	82 32	125	16 58
1000	86 15	1000	8 46

U

Let it be required, for example, to ascertain what effect will be produced upon a ray of light by four reflexions at the following angles.

Above the polarising angle   - $\left\{\begin{array}{cc} 77 & 40 \\ 70 & 9 \end{array}\right.$

Below the polarising angle   -   - $\left\{\begin{array}{cc} 50 & 26 \\ 35 & 18 \end{array}\right.$

The values of N and n in the Table corresponding to these angles are, 27, 6, 2, 10, and 64, and therefore we have

$$\frac{1}{2} + \frac{1}{6} + \frac{1}{2} + \frac{1}{10} = \frac{2604}{3240},$$

which being less than 1, the ray will not be polarised, but will require another reflexion either at 69° 1' or 41° 43', for the values of N and n corresponding to these angles are both

5 and $\frac{1}{5} + \frac{2604}{3240} = \frac{1626}{1620} = 1$ very nearly.

The formulæ in the preceding proposition are equally applicable to the second surfaces of transparent bodies, and to the separating surfaces of different media, $\frac{1}{m}$, and $\frac{m}{m'}$ being in these cases substituted in place of m.

<div align="center">PROP. XVIII.</div>

When a ray of light is once completely polarised, its polarisation will suffer no change, and the ray will preserve all its optical properties after any number of reflexions at any angle in the plane in which it was polarised, or after any number of refractions in a plane at right angles to that in which it was polarised.

If a ray is polarised by reflexion in the plane of the horizon, and is afterwards reflected at various angles in the plane of the horizon, one of the images will always vanish in the same position of the prism, as if it had not suffered a second

reflection. The same thing will happen if the direct ray is transmitted through a bundle of glass plates in which the plane of refraction is perpendicular to the horizon.

PROP. XIX.

When a polarised ray is incident at any angle upon a transparent body, in a plane at right angles to the plane of its primitive polarisation, a portion of the ray will lose its property of being reflected, and will entirely penetrate the transparent body. This portion of light, which has lost its reflexibility, increases as the angle of incidence approaches to the polarising angle, when it becomes a maximum.

A part of this Proposition constitutes one of the beautiful discoveries of MALUS, who found that at the polarising angle the second plate of glass " would no longer reflect a single " particle of light, either from its first or second surface."

The rest of the Proposition I have established by various experiments. In realgar, diamond, and oil of cassia, and in substances whose refractive power exceeds 1.600, the portion of light which *suffers reflexion* at the polarising angle is very considerable, and it will be seen from the Propositions in Sect. V., that if strong lights are used, there are no circumstances under which *every particle* of a beam of white light can lose its reflexibility.

PROP. XX.

If the reflecting plane upon which the polarised ray is received, is made to deviate in the slightest degree from the position which deprives the maximum portion of the ray of its reflexibility, a part of the light that had formerly lost its reflexibility will now suffer reflexion, and will be polarised in the plane of the SECOND REFLEXION, *whereas before the deviation took place, this portion of light was polarised in the plane of the first reflexion.*

When the plane of the second reflexion is perpendicular to the plane of primitive polarisation, every particle of the ray that suffers reflexion will be polarised in the last of these planes, but when the least deviation takes place, a part of the polarised ray is depolarised, so that it receives its character from the second reflexion, and is polarised in the plane of that reflexion. It is very interesting to observe two such opposite effects produced by the most minute change in the position of the second reflecting plane.

PROP. XXI.

If a ray of light reflected from a transparent body at any angle, excepting the polarising angle, is reflected from another body in a plane at right angles to that of its first reflexion, the reflected portion will be polarised by the second reflexion in the same manner, and at the same angle as if it had been direct light.

This proposition is deduced from experiment. The portion of light that is polarised at the first reflexion, will lose its reflexibility at the second reflexion, in proportion as the angle of reflexion approaches to the polarising angle.

PROP. XXII.

When a ray of light polarised by reflexion, is incident at any angle on the surface of a transparent body, so that the plane of the second reflexion is at right angles to the plane of the first reflexion, and suffers successive reflexions in the plane of the second reflexion, it will lose its reflexibility when it has undergone that number of reflexions which would have been necessary to polarise it, had it been direct light.

This result was at first deduced a priori from Prop. XVII. and XIX., and was afterwards established by experiment. The number of reflexions may be determined by the formulæ in Prop. XVII.

PROP. XXIII.

When a beam of light is emitted by the sun, or by any other body which does not shine by reflected light, the particles which compose it are in every state of POSITIVE *and* NEGATIVE *polarisation from particles completely polarised to particles not polarised at all.*

This Proposition is an expression of the experimental results in Prop. XVI. and XVII., and may be illustrated in the following manner, the terms *positive* and *negative* polarisation being employed to denote the two kinds of polarisation by reflexion and refraction at the polarising angle, or by reflexion in two opposite planes. A ray of direct light, before it is incident upon glass, may therefore be represented as consisting of a number of particles p, p, &c. of the following character.

$$+ \frac{p}{0^\circ}, \frac{p}{1^\circ}, \frac{p}{2^\circ}, \frac{p}{3^\circ}, \frac{p}{4^\circ}, \frac{p}{5^\circ}, \frac{p}{6^\circ} \cdots \cdots \frac{p}{56^\circ\ 45'}$$

$$- \frac{p}{0^\circ}, \frac{p}{1^\circ}, \frac{p}{2^\circ}, \frac{p}{3^\circ}, \frac{p}{4^\circ}, \frac{p}{5^\circ}, \frac{p}{6^\circ} \cdots \cdots \frac{p}{56^\circ\ 45'}$$

The particle $+$ $\frac{p}{0^0}$ will represent a particle so completely po-larised in a positive manner, that it will be polarised by re-flexion at 0^0 of incidence ; $-$ $\frac{p}{0^0}$ will represent a particle so completely polarised in a negative manner that it will be po-larised by reflexion in an opposite plane at 0^0 of incidence ; $+$ $\frac{p}{1^0}$ a particle so far polarised that it will require only a re-flexion at 1^0 of incidence to complete its polarisation, and so on with all the other particles till we come to $\frac{p}{56^0 \, 45'}$, which is a particle of direct light so completely unpolarised that it requires to be reflected at the maximum polarising angle before it can suffer complete polarisation.

This peculiar state of the rays before they fall upon a trans-parent body might have been deduced *a priori* by considering that when a mass of particles is projected from a self-lumi-nous body, the different sides of the rays, or poles of the luminous particles must have every possible position relative to the direction of their motion, which is the state described in the Proposition. If we break a tourmaline, for example, into a number of fragments, there will be a positive and a negative electrical pole in every possible direction, and a mass of moving tourmalines will have, nearly, the same relation to the tourmaline itself in which all the axes are regularly arrang-ed, as a beam of direct has to a beam of polarised light.

Corollary 1. A beam of light that has suffered reflexion at any angle above 0°, will be in a state which may be repre-sented in the following manner, the angle of incidence being supposed to be 4°.

Direct light $\quad + \frac{p}{0^\circ}, \frac{p}{1^\circ}, \frac{p}{2^\circ}, \frac{p}{3^\circ}, \frac{p}{4^\circ}, \frac{p}{5^\circ} \frac{p}{6^\circ} \cdots \frac{p}{56^\circ\,45'}$

Light reflected $\left.\begin{array}{c} \\ \end{array}\right\} + \frac{p}{0^\circ}, \frac{p}{0^\circ}, \frac{p}{0^\circ}, \frac{p}{0^\circ}, \frac{p}{0^\circ}, \frac{p}{z\,5^\circ}, \frac{p}{z\,6^\circ} \cdots \frac{p}{z\,(56^\circ\,45')}.$
at an angle of 4°

The negative part of the reflected beam may be represented in a similar manner. The particle of direct light $\frac{p}{0^\circ}$ being already polarised, will suffer no change by the reflexion (Prop. XVIII.) The particle $\frac{p}{1^\circ}$ being susceptible of polarisation by reflexion at an angle of 1° will also be polarised at any angle above 1°, and will therefore come into the state of $\frac{p}{0^\circ}$. In like manner the particles $\frac{p}{2^\circ}, \frac{p}{3^\circ}, \frac{p}{4^\circ}$ will all be polarised, and assume the state represented by $\frac{p}{0^\circ}$. The particle $\frac{p}{5^\circ}$ being susceptible of polarisation only at an angle of 5° or more will not be polarised at 4°, but will be brought into a state very near that of polarised light. It will therefore be represented by $\frac{p}{z\,5^\circ}$, z being a fractional coefficient, always less than 1, to be determined by the formulæ in Prop. XVII. For the same reason all the other particles will be brought into a state nearer that of perfect polarisation, and will be represented by $\frac{p}{z\,6^\circ}$ and $\frac{p}{z\,(56^\circ\,45')}$.

Cor. 2. In general, all the particles of a direct beam of light whose denominator is equal to or less than the angle of incidence will be brought by reflexion into the state of $\frac{p}{0^\circ}$, while all the particles whose denominator exceeds the angle of incidence will be brought into a state which may be found by multiplying their state in the direct beam by $\frac{1}{z}$, z being determined by Prop. XVII.

SECT. V. *On the nature and origin of the apparently unpolarised light which exists at the maximum polarising angle.*

I have already shown in a former paper,* that in substances of a high refractive power, such as *realgar* and *diamond*, there is a great quantity of apparently unpolarised light reflected at the polarising angle, so that the image which would have vanished by the application of calcareous spar, had the reflexion been made from *water* or *crown glass*, possessed in these cases a considerable brilliancy. The comparative intensity of the light of this image is indeed so great, that only a very small portion of the incident light seems to be polarised. I was at first much surprised at witnessing this phenomenon, as I had been led to believe from the first Memoir of MALUS, that one of the pencils formed by calcareous spar vanished when the light was reflected from all other bodies as well as from *water* and *glass*. There is some reason to think, however, that MALUS afterwards observed the same fact, for in a subsequent Memoir, he makes use of the term *maximum polarising angle*, in which the knowledge of it seems to be implied.

The extreme difficulty of accounting for such an unexpected phenomenon, probably deterred him from even mentioning the subject in any of his Memoirs ; and I am not ashamed to avow that the investigation of this point alone has cost me more labour than any other branch of the polarisation of light. The existence of a quantity of apparently unpolarised light, in a pencil reflected at the polarising angle, appeared completely paradoxical, and it was obvious that no satisfactory generalisation of the phenomena could be given while this difficulty remained unsolved.

* See Phil. Trans. for 1814, Part I. p. 230.

I at first imagined that bodies with a high refractive power approached to the metals in their mode of action upon light, but this conjecture was refuted by experiments which showed that the pencil did not consist of two oppositely polarised portions. When I discovered the law of successive reflexions, as stated in Prop. XVI. and XVII., the difficulty of explaining the phenomenon seemed to increase. What MALUS would have called the unpolarised portion of a beam of light, reflected at 62° from glass, was now shown to be so far polarised, that its polarisation was completed by a second reflexion at the same angle, so that it became still more improbable that unpolarised light could exist at the polarising angle itself. All these difficulties, however, were immediately removed by the discovery of the law of the tangents, and of the polarisation of the differently coloured rays at angles of incidence depending on their respective indices of refraction. The explanation which now suggested itself was confirmed by experiment, and I was thus led after much fruitless investigation to the results expressed in the following Propositions.

PROP. XXIV.

If a pencil of white light is incident at the maximum polarising angle upon any transparent body whatever, a portion of the reflected pencil, consisting of the mean refrangible rays, will be completely polarised, while another portion of the beam, consisting of the blue and red rays, will not be completely polarised, and will therefore not vanish when the image from which the light proceeds is examined with a prism of calcareous spar.

It is obvious from Prop. II. that all the rays which compose a beam of white light cannot be polarised at the same angle

of incidence.   When the pencil is incident at the maximum polarising angle, or at an angle whose tangent is equal to the index of refraction for the mean refrangible rays, these rays alone will be polarised.   Neither the *red* rays, which are incident at an angle above their polarising angle, nor the blue rays which are incident at an angle below their polarising angle, will be completely polarised ; and when the reflected pencil which contains them is viewed through a doubly refracting crystal, the *mean* refrangible rays will vanish, while the *red* and *blue* rays will compose a beam nearly white, and will not vanish, in consequence of its not being completely polarised.

PROP. XXV.

If a pencil of WHITE *light polarised by reflexion is incident at the polarising angle upon any transparent surface, so that the plane of the second reflexion is at right angles to the plane of its primitive polarisation, a portion of the pencil consisting of the mean refrangible rays will lose its reflexibility, and will entirely penetrate the second surface, while another portion of the beam, composed of the blue and red rays, will not lose its reflexibility, but will suffer reflexion and refraction like ordinary light.*

This proposition founded also on experiment may be proved by the same reasoning as the preceding, for since the angle at which polarised rays lose their reflexibility is the same as the angle at which they are polarised, only one set of the rays which compose a white beam can lose their reflexibility at the same angle.

PROP. XXVI.

The imperfectly polarised portion of light described in Prop. XXIV.
and the portion which does not lose its reflexibility as described
in Prop. XXV., increase with the dispersive and the refractive
power of the reflecting surface, so that in substances where the
dispersive and refractive forces are very great, these portions
constitute in the one case almost the whole of the reflected pencil,
and in the other almost the whole of the pencil that would have
been reflected under ordinary circumstances.

When the dispersive power of the reflecting surface is so
high as to throw the blue and red rays to a great distance
from the mean ray, the quantity of polarised light at the mean
polarising angle must be very small, and must obviously di-
minish as the dispersive power increases, the quantity of im-
perfectly polarised light consisting of the blue and red rays
increasing in the same proportion. When the refractive
power is high, the polarising angle increases, and the quan-
tity of reflected light becomes very great, being, in the case
of diamond, about one half of the incident beam. Hence in
rock crystal, which has a higher refractive power, and a lower
dispersive power than water, the image does not wholly
vanish at the polarising angle.

PROP. XXVII.

If a pencil of HOMOGENEOUS *or coloured light is incident upon
any transparent body at an angle whose tangent is equal to
the index of its refraction, every ray of the reflected pencil will
be completely polarised in the plane of reflexion.*

This proposition is a necessary consequence of those which
precede it; and I have also established it by direct experi-
ments upon diamond and realgar.

PROP. XXVIII.

If a pencil of HOMOGENEOUS *or coloured light is incident under
the circumstances described in Prop. XXV., every ray of it will
lose its reflexibility.*

This proposition, which is also deducible from those which
precede it, has been established by experiment.

PROP. XXIX.

If a beam of WHITE *light suffers more than one reflexion, every ray
of it will be completely polarised when the angles of incidence are
of such a magnitude that the sum of the terms of the formulæ
given under Prop. XVII. is equal to* 1, *the index of refrac-
tion for the extreme red ray being substituted in place of* m
*if the angles are above the polarising angle, and the index of
refraction for the extreme blue ray if the angles are below the
polarising angle.*

This Proposition is manifestly deducible from Prop. XVII.
compared with Prop. XXVII. Calling *dm* the part of the

whole refraction to which the dispersion, or the distance be-
tween the extreme rays is equal, the formulæ will become

$$\left(\frac{1}{\frac{\text{tang. A.}}{m-\frac{1}{2}dm}}\right)^3 + \left(\frac{1}{\frac{\text{tang. A}'}{m-\frac{1}{2}dm}}\right)^3 + \left(\frac{1}{\frac{\text{tang. A}''}{m-\frac{1}{2}dm}}\right)^3 = 1$$

$$\left(\frac{1}{\frac{m+\frac{1}{2}dm}{\text{tang. }a}}\right)^3 + \left(\frac{1}{\frac{m+\frac{1}{2}dm}{\text{tang. }a'}}\right)^3 + \left(\frac{1}{\frac{m+\frac{1}{2}dm}{\text{tang. }a''}}\right)^3 = 1$$

If the angles are partly above, and partly below the pola-
rising angle, for example, at the angles A, a, a' A'' then the
formula will become

$$\left(\frac{1}{\frac{\text{tang. A}}{m-\frac{1}{2}dm}}\right)^3 + \left(\frac{1}{\frac{m+\frac{1}{2}dm}{\text{tang. }a}}\right)^3 + \left(\frac{1}{\frac{m+\frac{1}{2}dm}{\text{tang. }a'}}\right)^3 + \left(\frac{1}{\frac{\text{tang. A}''}{m-\frac{1}{2}dm}}\right)^3 = 1$$

SCHOLIUM.

I have determined the values of dm for 151 different sub-
stances, and have published a Table containing 137 of these
in my *Treatise on New Philosophical Instruments*, p. 315. The
value of m or the mean index of refraction, was found in the
common way by measuring the angle of deviation produced
by a prism of the substance under examination. The values
of dm were computed, from measures taken with a new in-
strument, by means of a formula investigated by BOSCOVICH,
and used in the reduction of all his valuable experiments.
This formula requires that the ray should be incident perpen-
dicularly upon the first surface, but it will be found in prac-
tice that the dispersion of the prism under examination is
equally corrected by the standard prism, when the ray is
incident several degrees on either side of the perpendicular.

I have thus endeavoured as briefly as possible, and perhaps

more briefly than a new subject required, to give an account
of the experiments and reasonings by which I have established
the laws of the polarisation of light by reflexion from trans-
parent bodies. These experiments have been extended to
other branches of this subject, and in subsequent Memoirs
I shall take the liberty of soliciting your attention to the laws
which regulate the polarisation of light by *transmission through
uncrystallized plates;*—by *reflexion from metallic and oxidated
surfaces*; and by the *separation of light into two pencils by the
action* of regularly crystallized bodies. In the investigation of
the properties of metallic and oxidated surfaces, my experi-
ments have been attended with the most successful results.
I have discovered that the beautiful *complementary colours* pro-
duced by the action of crystalline bodies upon polarised light,
are exhibited under singular circumstances by reflexions from
silver and *gold*, and to a certain degree from *other metals;*—
and that some metallic bodies have the power of polarising
a beam of light in the plane of incidence by six or seven suc-
cessive reflexions, while other metals are not able to polarise
it even after twenty or thirty successive reflexions.

In these enquiries I have made use of no hypothetical as-
sumptions. In imitation of MALUS, the language of theory
has been occasionally employed, but the terms thus introduced
are merely expressive of experimental results, and enable
us to avoid frequent and perplexing circumlocutions. The
science of physical optics is not yet in such a state as to autho-
rise the construction of a new nomenclature. When disco-
very shall have accumulated a greater number of facts, and
connected them together by general laws, we may then safely
begin to impose better names, and to speculate respecting the
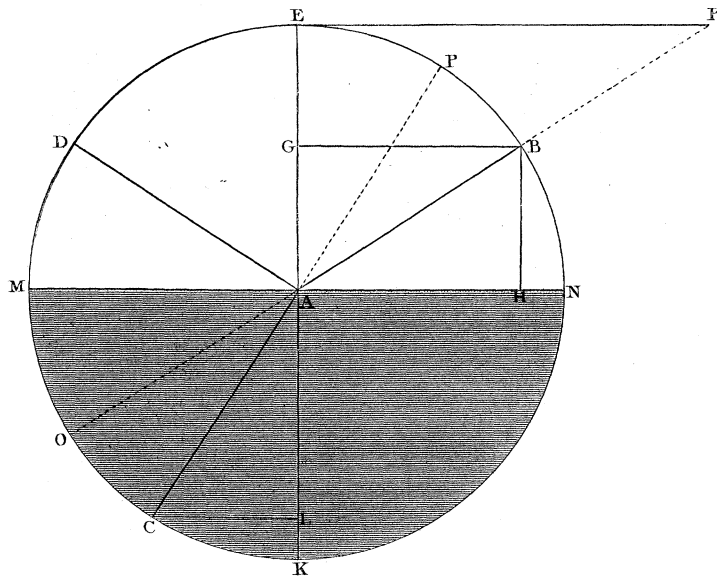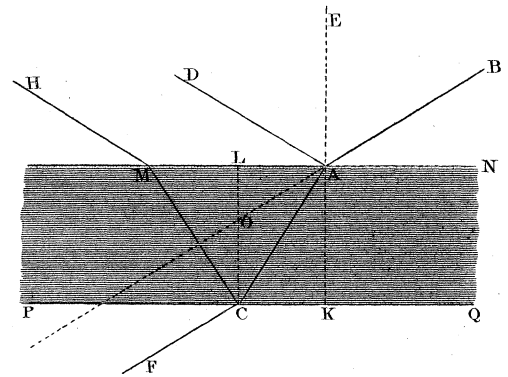
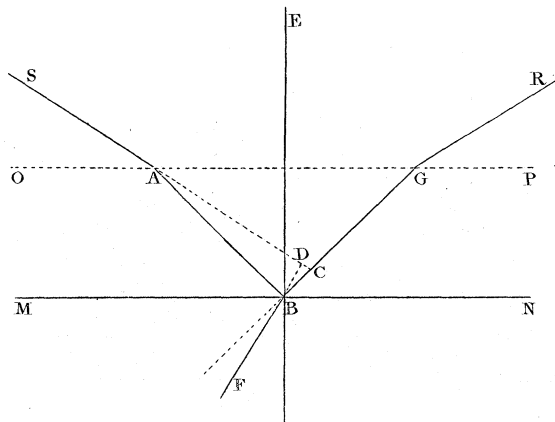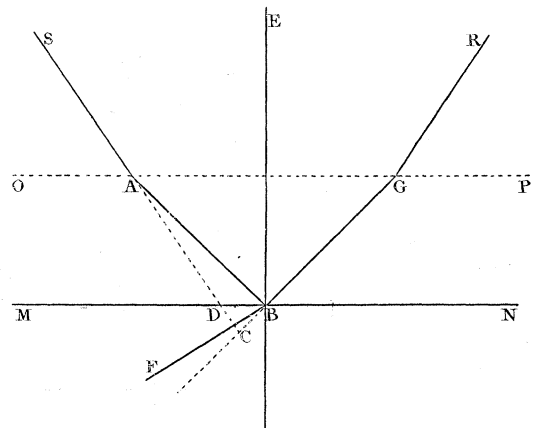*Fig. 1.*

Fig. 3.

Fig. 2.

Fig. 4.

cause of those wonderful phenomena which light exhibits under all its various modifications.

In the preceding pages, I have more than once had occasion to establish conclusions opposite to those which MALUS had deduced from less numerous experiments; and indeed the whole of this paper is founded on relations which he believed to have no existence. In differing, however, from this eminent philosopher, I trust I have always done it with that respect which it is impossible not to feel for his character and labours. It has fallen to the lot of few to enrich science with so many new and striking discoveries, and if he has failed in pursuing them through all their consequences, we must ascribe it to the limited interval which he was allowed to devote to science, and to the influence of that cruel disease which terminated so prematurely his short but brilliant career. Those, who without repeating his experiments endeavoured during his life to depreciate his labours, are alone capable of wounding his memory. Those who, like him, have pursued science under the oppression of bodily suffering;—who have been instructed and delighted with his discoveries, and who have patiently followed him in the path of research, will feel it their truest pride to do justice to his memory, and will never be able to review his labours without mingling their sorrow with their admiration.

<div align="center">I have the honour to be, &c.</div>

<div align="right">DAVID BREWSTER.</div>

Edinburgh, February 11, 1815.